Phase-Noise Analysis of MEMS-Based Circuits and Phase Shifters

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***Abstract*—The effect of Brownian, acceleration, acoustic, and power-supply noise on Microelectromechanical system (MEMS)-based circuits has been calculated for MEMS-based circuits (phase shifters, delay circuits). The calculations are done for capacitive shunt MEMS switches and metal-to-metal contact series MEMS switches. It is found that these effects result in both an amplitude and phase noise, with the phase noise being around 100× larger than the amplitude noise. The phase noise due to Brownian motion is negligible for MEMS switches with $k \simeq 10$ N/m, $g_0 > 2$ μm, $Q > 0.5$, and $f_0 \simeq 50$ kHz. The effect of acceleration and acoustic noise is negligible for a total acceleration noise of 10 g or less and a total acoustic noise of 74-dB sound pressure level. The power-supply noise depends on the bias conditions of the MEMS element, but is negligible for MEMS switches with a bias voltage of 0 V and a total noise voltage of 0.1 V or less. It is also found that metal-to-metal contact series switches result in much less phase noise than standard capacitive shunt switches. The phase noise increases rapidly for low spring-constant bridges ($k = 0.24$ N/m), low-height bridges, and bridges with a large mechanical damping ($Q < 0.3$). Also, varactor-based designs result in 30–40 dB more phase noise than switch-based circuits. This paper proves that microwave passive circuits built using MEMS switches (with a proper mechanical design) can be used in most commercial and military applications without any phase-noise penalty.**

I. Introduction

MICROELECTROMECHANICAL system (MEMS) series and shunt switches have been recently used in many low-loss phase-shifter circuits [1]–[6]. The MEMS phase shifters are typically implemented using standard p-i-n diode or FET switch designs, except that the switching element is replaced by a MEMS switch. MEMS switches result in 0.05–0.2-dB loss at 1–100 GHz and, therefore, yield excellent performance up to 120 GHz. However, MEMS switches suffer from Brownian noise motion, which is due to the thermal energy stored in the system. The Brownian noise results in a random change in the capacitance of the switch, which, in turn, results in an additional phase and amplitude noise at the output of a MEMS-controlled oscillator or phase shifter. The effect of the Brownian noise on oscillators has been derived and demonstrated by Young and Boser [7] and Dec and Suyama [8], [9]. The goal of this paper is to calculate the Brownian noise effect on phase shifters and to determine the physical parameters that need to be controlled in the MEMS structure so as to result in very low additional noise.

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II. Review of Brownian Noise

The Brownian noise of a mechanical structure with a spring constant k, a damping factor b, and a mechanical resonant frequency ω_0 has been derived by Gabrielson [10] and is

$$x_n = \frac{\sqrt{4k_BTb}}{k} \frac{1}{1-(\omega'/\omega_o)^2 + j\omega'/Q\omega_o} \left(\text{m}/\sqrt{\text{Hz}}\right) \quad (1)$$

where k_B is the Boltzman constant, $Q = k/(\omega_0 b)$ is the quality factor of the MEMS bridge, and $\omega_0 = \sqrt{k/m}$, where m is the mass of the bridge. The mechanical force acting on the bridge due to the thermal noise is $f_n = \sqrt{4k_BTb}$ (in N/$\sqrt{\text{Hz}}$) and is equivalent to the noise voltage from a resistor R, $v_n = \sqrt{4k_BTR}$ (in V/$\sqrt{\text{Hz}}$), with b replaced by R. The damping factor b is dependent on the height of the MEMS bridge, the number of holes in the bridge, and the viscosity of the suspending medium (air, nitrogen, etc.) [10], [11]. The damping factor also has a strong effect on the switching time if the resulting mechanical Q is less than 0.5. In practice, it is good to design for a $0.5 \geq Q \leq 5$ so as to result in a switching time that is limited by the mechanical characteristics (mass, spring constant, etc.) of the switch and not by the damping of the medium underneath the switch.

It can be seen that switches with large spring constants and low damping coefficients result in very low Brownian noise. The Brownian noise of a MEMS bridge with $k = 10$ N/m, $f_0 = 50$ kHz, $T = 300$ K, and $Q = 0.3, 1, 3$ are plotted in Fig. 1. If the spring constant is reduced to 1 N/m, the values on Fig. 1 will increase by 10 dB. It is seen that the low-frequency value of the Brownian noise is of the order of 10^{-14} m/$\sqrt{\text{Hz}}$ for a standard MEMS bridge. The Brownian noise decreases very quickly after the mechanical resonant frequency and is insignificant after $5\omega_0$.

The Brownian noise can be represented as a summation of sinusoidal waveforms with random amplitude and phase. A single sinusoid with a mechanical frequency of ω' and amplitude equal to the square root of noise power in a 1-Hz bandwidth around ω' is written as

$$x_n = \sqrt{2\overline{x_n^2}(\omega')}\sin(\omega' t) \left(\text{m}/\sqrt{\text{Hz}}\right). \quad (2)$$

III. Brownian Noise Effects for MEMS Shunt Switches

Consider the MEMS shunt switch shown in Fig. 2. The switch could be the standard capacitive shunt design developed by Raytheon with a center pull-down electrode [2] or the dc-contact shunt switch with two pull-down electrodes developed by Muldavin *et al.* [12] and Feng *et al.* [13]. Alternatively,

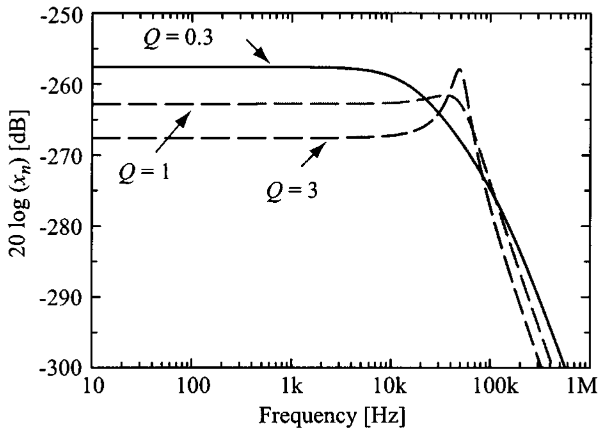

Fig. 1. Calculated Brownian noise component of a MEMS element with $k=10$ N/m, $f_0=50$ kHz, $T=300$ K, and $Q=0.3$, 1, 3.

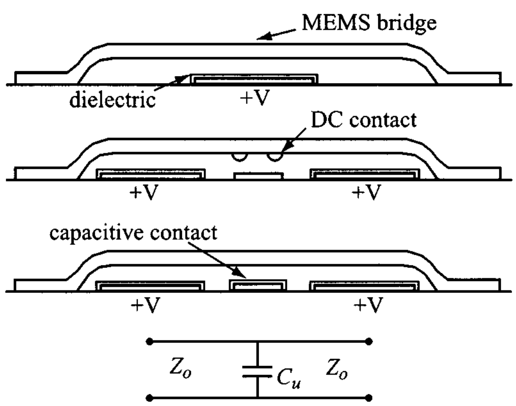

Fig. 2. Different shunt switch topologies (capacitive, dc-contact or capacitive with two-pull down electrodes), all with an up-state capacitance C_u.

it could also be a low spring-constant switch [4]. What is important is that when the switch is in the up-state position, it allows the microwave energy to pass with very little insertion loss, and when the switch is in the down-state position, it presents a short circuit (or near short circuit) to ground and reflects all the incident energy at the design frequency.

The up-state capacitance of the shunt switch is given by $C_u = C_{\rm pp} + C_f$, where C_f is the fringing capacitance and $C_{\rm pp} = \epsilon A/g_0$ is the parallel-plate capacitance. It is customary to take $C_f = \gamma C_{\rm pp}$ and $\gamma < 1$ for most designs. When Brownian noise is included, the parallel-plate capacitance can be written as

$$C_{\rm pp} = \frac{\epsilon A}{g_0 + x_n} \simeq C_{\rm pp}\left(1 - \frac{x_n}{g_0}\right) \tag{3}$$

where x_n is the Brownian noise movement of the MEMS bridge. The up-state capacitance of the bridge is then

$$C_u = C_{u0}\left(1 - \frac{1}{1+\gamma}\frac{x_n}{g_0}\right) \tag{4}$$

where $C_{u0} = C_f + C_{\rm pp} = (1+\gamma)C_{\rm pp}$ is the up-state capacitance with zero Brownian noise. The up-state capacitance of the shunt switch results in a scattering parameter of

$$S_{21} = \frac{1}{1 + \left(j\omega C_u Z_0/2\right)} \tag{5}$$

where ω is the operating frequency and Z_0 is the t-line impedance.

A. Phase Noise

The phase of S_{21} above is the phase delay due to the up-state capacitance of the MEMS switch and is

$$\phi = \frac{-\omega C_u Z_0}{2}. \tag{6}$$

For $C_{u0} = 35$–70 fF and $f = 10$ GHz, the phase delay is $\phi_0 = 0.055$–0.11 rad. Using (4), the phase delay with Brownian noise can be written as

$$\phi = \phi_0\left(1 - \frac{1}{1+\gamma}\frac{x_n}{g_0}\right). \tag{7}$$

If a microwave signal with a representation of $A\cos(\omega t)$ is incident on the MEMS bridge, the output signal is (neglecting for now, any amplitude change)

$$V_o(t) = A\cos(\omega t + \phi) = A\cos\left(\omega t + \phi_0\left(1 - \frac{1}{1+\gamma}\frac{x_n}{g_0}\right)\right). \tag{8}$$

Using (1), the output signal can be expanded to be

$$V_o(t) = A\cos(\omega t + \phi_o) + \frac{1}{(1+\gamma)}\frac{A\phi_o}{2g_o}\sqrt{2\overline{x_n^2}} \cdot\Big[\cos\left[(\omega - \omega')t + \phi_o\right] - \cos\left[(\omega + \omega')t + \phi_o\right]\Big]. \tag{9}$$

The power in each of the sidebands relative to the carrier is the additional phase noise due to the MEMS bridge and is

$$P_{\rm ph} = \frac{1}{2}\frac{1}{(1+\gamma)^2}\frac{\overline{x_n^2}}{g_0^2}\phi_0^2\,/\mathrm{Hz} \tag{10}$$

and at low mechanical frequencies ($\omega' < \omega_0$), $\overline{x_n^2} = (4k_BTb)/k^2 = \overline{f_n^2}/k^2$ (in m^2/Hz), where f_n is the mechanical force due to thermal noise. For $C_u = 50$ fF, $\gamma = 0.35$, $f = 10$ GHz, $Z_0 = 50\ \Omega$, and a MEMS bridge parameters given by $k = 10$ N/m, $Q = 1$, $f_0 = 50$ kHz, and $g_0 = 3\ \mu$m, the phase noise relative to the carrier is calculated to be $P_{\rm ph} = 10^{-18}$ (-180 dBc/Hz) for a single MEMS bridge. If the signal power is 1 mW, the phase noise is much lower than the thermal noise, which is -174 dBm/Hz (noise figure $= 3$ dB).

There are a four points worth mentioning. The first is the effect of the spring constant on the phase noise (k^{-2}). If the spring constant drops 1 N/m, the phase noise increases to -160 dBc/Hz for a single MEMS bridge. The second point is the strong dependence of the phase noise on the height of the MEMS structure. Equation (10) shows that the phase noise depends on the factor b/g_0^2. However, for a fixed–fixed circular or rectangular plate, b depends on g_0^{-3} [14] and, therefore, the phase noise depends on g_0^{-5}. If a structure with $g_0 = 3$ μm is lowered to $g_0 = 1.2$ μm, the phase noise will increase by 20 dB.

The third point is that the phase noise is constant at low offset frequencies and falls quickly after the mechanical resonant frequency (30–300 kHz, depending on the design). However, for most radar applications with moving target indicators, the final frequencies of interests are in the 5–100-kHz range. Designing a MEMS structure with a low mechanical resonant frequency (5–20 kHz) will not necessarily result in better performance since this structure will generally have a low spring constant and result in a higher low-frequency phase noise. Also, the switching time will be substantially lower in this case.

Finally, (6) and (10) seem to indicate that the phase noise is dependent on f^2. This is not entirely correct for narrow-band systems. If a circuit is well designed, then $\phi_0 = (-\omega C_{u0} Z_0)/2$ should be chosen to be the same value at any operation frequency. In other words, C_{u0} at $f = 60$ GHz should be 6× smaller than C_{u0} at 10 GHz. Therefore, the phase-noise component is independent of the design frequency.

B. Amplitude Noise

The magnitude of S_{21} in (5) is

$$|S_{21}| \simeq 1 - \frac{\omega^2 C_u^2 Z_0^2}{8}. \tag{11}$$

The effect of the Brownian noise on the amplitude of the output signal is

$$V_o(t) = \left(1 - \frac{\omega^2 C_u^2 Z_0^2}{8}\right)(A\cos\omega t). \tag{12}$$

Substituting C_u and x_n using (2) and (4), we get

$$V_o(t) = A\cos(\omega t) + \frac{1}{(1+\gamma)}\frac{A\phi_o^2}{2g_o}\sqrt{2\overline{x_n^2}} \cdot \Big[\sin(\omega - \omega')t - \sin(\omega + \omega')t\Big]. \tag{13}$$

The power in each of the sidebands relative to the carrier is the additional amplitude noise due to the MEMS structure and is

$$P_{\rm am} = \frac{1}{2}\frac{1}{(1+\gamma)^2}\frac{\overline{x_n^2}}{g_0^2}\phi_0^4 \,/\text{Hz}. \tag{14}$$

For the case outlined above, $P_{\rm am} = P_{\rm ph}/162$ and is, therefore, negligible.

IV. Phase-Noise Reduction Using N MEMS Shunt Switches

If the shunt switch with a capacitance C_u is fabricated using N MEMS shunt switches placed next to each other, each with a capacitance of C_u/N, the phase noise becomes

$$P_{\rm ph}(\omega') = \frac{1}{2}\frac{1}{N}\frac{1}{(1+\gamma)^2}\frac{\overline{x_n^2}(\omega')}{g_0^2}\phi_0^2 \,/\text{Hz} \tag{15}$$

where ϕ_0 is the phase delay due to the total capacitance C_u and x_n is the Brownian noise movement of each MEMS switch with a reduced geometry. The phase noise is reduced by N as compared to a single switch since the noise components from the different MEMS switches are not coherent. This is correct only if the mechanical damping of the small MEMS switches is the same as the damping of the large MEMS switch (with capacitance C_u). In general, the damping of MEMS structures (or switches) with no holes is proportional to $(wl)^2$, where l and w are the length and width of the MEMS switch. Therefore, the reduction in the phase noise will be N^3, with the additional factor N^2 due to the reduced damping component of each MEMS switch with a width w/N. The above is valid for $N = 3$–6 and then the bridge becomes very narrow and the damping equations with b proportional to w^2 are not accurate anymore. In the case of holes in the MEMS bridge, the damping coefficient is more complicated, and one must obtain the new value of b for the reduced-width geometry. Still, there is at least a reduction by N as compared to a single switch. This is important for low-k switches and varactor-based phase shifters (see Section VI).

V. Phase Shifters Based on Shunt Switches

Phase shifters use a large number of MEMS switches. The Brownian noise of different shunt capacitive switches are not correlated and, therefore, it is easy to prove that phase-noise power of multiple shunt switches is the sum of the phase-noise powers resulting from each individual switch. This is the same for the total noise in a resistor network. If the MEMS switch is in the up-state position, it contributes a phase-noise component to the output signal. However, if the MEMS switch is in the down-state position, it is fixed and does not contribute any phase noise.

Case 1: Switched-Network Designs: A switched network phase shifter requires four shunt switches per bit [see Fig. 3(a)] and the signal will always pass by two MEMS switches per bit, depending on the switch positions. For a 3-bit design, the output phase noise is 6× higher than the phase noise of a single MEMS bridge. If the switched network is a simple transmission line, then it is possible to use three switches per bit [15], and the phase noise will be 3×–6× higher than the phase noise of a single MEMS bridge, depending on the switch selection.

Case 2: Reflect-Line Designs: The phase noise of a reflect-line phase shifter depends on the switch selection. In the case of a delay of 0°, the first set of MEMS switches are pulled down and there is no added phase noise. In the case of the longest delay, the signal passes *twice* by several MEMS bridges. For a 2-bit design, the longest delay necessitates three MEMS bridges and the total phase noise is, therefore, 6× higher than the phase noise of a single shunt switch.

Case 3: Reflect-Line With 3-dB Couplers: In this case, the input signal (the carrier) is divided into two parts, each with half of the power and phased 0°/90° apart [see Fig. 3(b)] [16]. The phase noise generated in *each* arm of the 3-dB coupler is the same as above in Case 2, but is referenced to one-half of the signal power. Also, the phase-noise components from each arm of the 3-dB coupler do not add in phase at the output port

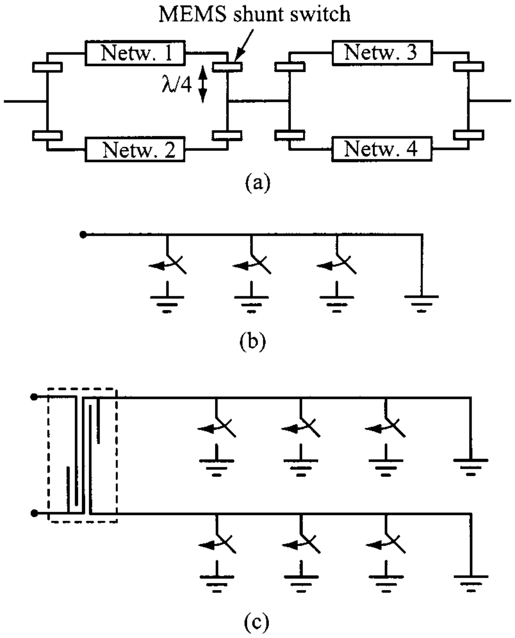

Fig. 3. (a) Phase shifter based on switched networks. (b) Switched delay lines. (c) Reflect-line with a 3-dB coupler.

and, therefore, the total noise power from both arms is divided equally between the input and output ports. If the input port is well matched, then the output phase noise relative to the carrier is just one-half that of Case 2.

If several 3-dB couplers are connected in series to build, for example, a 4-bit phase shifter from two 2-bit phase shifters, then the phase noise at the output port will be the addition of the phase noise resulting from each of the 3-dB coupler/delay-line units. This holds true, of course, if the units are well matched to each other.

VI. Phase Shifters Based on MEMS Varactors

The reflection phase from a capacitive load is

$$\phi = 180^\circ + 2\tan^{-1}\frac{|X|}{Z_0}, \qquad X \leq 0 \tag{16}$$

and the phase shift is achieved by varying the capacitance value using a bias voltage ($X_1 < X < X_2$) with $X_1 = -j/(\omega C_{\min})$, $X_2 = -j/(\omega C_{\max})$. In a good varactor design, $X_1 = -j80\ \Omega$ to $-j100\ \Omega$, and $X_2 = -j20\ \Omega$ to $-j25\ \Omega$ for a capacitance ratio of 4 : 1. It has long been recognized that a much larger phase shift can be obtained with the same capacitance ratio if an inductance is placed in series with the varactor. The value of the inductance is chosen so as to resonate with the average value of the varactor reactance. This results in the largest phase angle swing around the $X = 0$ location on the Smith chart and, therefore, the largest phase change for the same capacitance ratio (Fig. 4). It is possible to attain a phase change of 120° with a capacitance ratio of 4 : 1 using this technique [17], [18].

The area around the $X = 0$ loci is also the most sensitive to variation in the load reactance, and it is at this point that we will

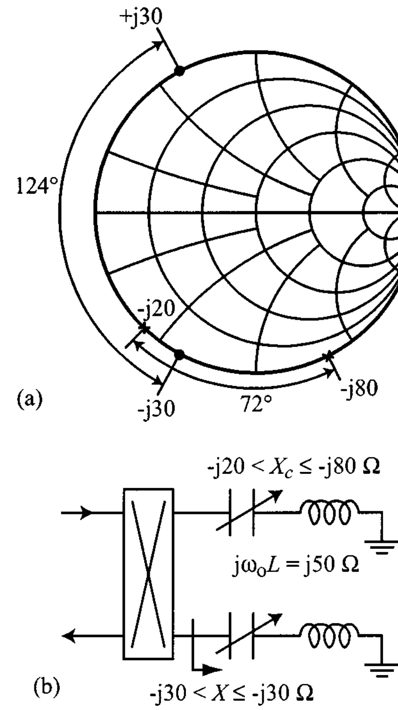

Fig. 4. (a) Reflection phase from a capacitive load. (b) Inductively tuned capacitive load with a 3-dB coupler.

calculate the added phase noise due to the varactor Brownian noise. In this case, the total reactance is

$$|X| = \omega L - \frac{1}{\omega C}, \qquad X \simeq 0 \tag{17}$$

where C is the varactor capacitance that results in a reactance of $|X_{\rm av}| = 1/\omega C = (X_1 + X_2)/2$ ($|X_{\rm av}| \simeq 50\ \Omega$). Substituting C using (4), we get

$$|X| = \omega L - \frac{1}{\omega C} - \frac{1}{\omega C}\frac{1}{1+\gamma}\frac{x_n}{g_0} \simeq \frac{1}{1+\gamma}|X_{\rm av}|\frac{x_n}{g_0}. \tag{18}$$

Using (15) and the small-signal expansion of $\tan^{-1}(x)$, this results in a reflection phase of

$$\phi = 180^\circ - 2\frac{1}{1+\gamma}|X_{\rm av}|\frac{x_n}{g_0} = 180^\circ + \Delta\phi. \tag{19}$$

The signal, when reflected from the varactor, will have the form $V_o(t) = A\cos(\omega t + 180^\circ + \Delta\phi)$. When this is expanded and x_n is replaced using (1) and (2), the phase noise relative to the carrier is found to be

$$P_{\rm ph} = \frac{1}{(1+\gamma)^2}\frac{2|X_{\rm av}|^2}{Z_0^2}\frac{\overline{x_n^2}}{g_0^2}\ /\text{Hz}. \tag{20}$$

For $|X_{\rm av}| = 50\ \Omega$, and for the case outlined above ($k = 10$ N/m, $g_0 = 3\ \mu$m, $f_0 = 50$ kHz, $\gamma = 0.35$, $Q = 1$), the added phase noise is 6.4×10^{-16}/Hz (−152 dBc/Hz) and is much larger than the noise due to the standard MEMS shunt switches. This is due to the fact that the entire MEMS bridge capacitance is used in the design of the phase shifter. Again, the phase noise is dependent on k^{-2} and g_0^{-5}, and for low spring-constant varactors ($k = 1$ N/m), which are suspended 1.5 μm above the bottom

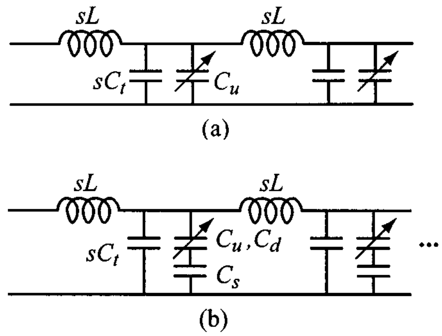

Fig. 5. (a) Equivalent circuit of a distributed analog and (b) digital phase shifter.

electrode, the phase noise increases to −117 dBc/Hz close to the carrier (up to the mechanical resonant frequency). This phase noise is not acceptable in many radar applications.

If the phase shifter is based on a 3-dB coupler design with MEMS varactors, then the total noise at the output port will be one-half the value of a single capacitive load (following the same reasoning as above). If one uses four 3-dB couplers/varactors units connected in series to achieve a phase shift of 360°, then the total phase noise of the phase shifter will be two times the value of a single capacitive load.

VII. T-Line Circuits Using MEMS Varactors

A MEMS varactor can also be used in a t-line circuit in the transmission mode for tuning purposes (matching networks, variable delay, etc.). In this case, (6) applies, except that ϕ_0 will generally be much larger than 0.1 rad ($\phi_0 = 0.5$–1.2 rad). Also, this circuit can result in a large reflection coefficient, and must be tuned using series inductors for $|S_{21}| \simeq 0$ dB. The phase noise derived in (11) also applies, but is 20–30 dB larger due to the higher shunt capacitance. The phase noise adds incoherently if several MEMS varactors are used.

VIII. Distributed Phase Shifters

Distributed microelectromechanical system transmission lines (DMTL) were first introduced by Barker and Rebeiz for applications in wide-band phase shifters and high-isolation switches [19], [20]. The idea is to suspend a periodic MEMS varactor over a t-line, and by applying a single control voltage, the height of the varactors can be changed [see Fig. 5(a)]. This, in turn, results in an increase in the loading capacitance and, therefore, in a decrease in the phase velocity of the line and a true-time delay phase shifter. In this case, the phase shift is continuously controlled using a single analog voltage. Using this approach, a dc–120-GHz phase shifter was demonstrated with an insertion loss of 5 dB for 360° of phase shift at 75–110 GHz [20].

Analog Design: The phase delay *per section* for a periodically loaded t-line is [19]

$$\phi = \omega\sqrt{sL_t(sC_t + C_u)} \tag{21}$$

where s is the period in centimeters, L_t and C_t are the inductance and capacitance per unit length of the unloaded t-line, and C_u is the MEMS bridge loading capacitance in F. If C_u is replaced by (4), the phase becomes

$$\phi = \phi_0 \left(1 - \frac{1}{2}\frac{1}{1+\gamma}\frac{C_u}{sC_t + C_u}\frac{x_n}{g_0}\right) = \phi_0 + \Delta\phi. \tag{22}$$

Note that ϕ_0 is the phase delay per section with no Brownian noise and is not the phase shift per section due to the change in the bridge capacitance.

Equation (21) is very similar to (7), except with the additional factor of $C_u/2(sC_t + C_u)$. Notice that C_{u0} is not employed since the up-state capacitance depends on the bias voltage and the position of the MEMS bridge. In order to predict the worst phase-noise performance, the up-state capacitance should be taken at the largest bias voltage (and smallest gap over the line). The resulting single-sideband phase noise relative to the carrier *per section* can be calculated to be

$$P_{\rm ph} = \frac{1}{4}\frac{1}{(1+\gamma)^2}\left(\frac{C_u}{sC_t + C_u}\right)^2 \frac{\overline{x_n^2}}{g_0^2}\phi_0^2\,/\text{Hz}. \tag{23}$$

A well-designed analog DMTL phase shifter has a maximum loading capacitance of $C_{u\,\max} \simeq 3(sC_t)$ on a quartz substrate and uses 16–20 periodic sections to achieve a phase shift of 180° at maximum bias [19], [20]. The phase noise at the output of the DMTL phase shifter adds linearly with the number of sections. For a coplanar waveguide (CPW) line on quartz with $Z_u = \sqrt{L_t/C_t} = 100\ \Omega$, $\epsilon_r = 3.9$, $s = 900\ \mu$m, and $C_{u\,\max} = 3(sC_t)$, the phase delay per section at 10 GHz is $\phi_0 = 33°$. The height of the MEMS varactor is typically 2 μm at maximum bias. For a MEMS bridge with $k = 10$ N/m, $\gamma = 0.35$, $f_0 = 50$ kHz, $Q = 1$, and $g_0 = 2\ \mu$m, the phase noise is −164 dBc/Hz *per section*. The phase shift for a 180° phase shift is 16×–20× higher depending on the design (around −152 dBc/Hz).

It is important to note that phase noise does exist at zero bias (and zero phase shift). In this case, the bridge height is around 3 μm and there is less loading and less phase delay per section since $C_u \simeq 2(sC_t)$. The resulting phase noise per section is around −169 dBc/Hz. Of course, the calculated phase noise depends on g_0^{-5} and k^{-2}, as mentioned above.

Digital Designs: The periodic MEMS varactor can be made to switch between two capacitance values [see Fig. 6(b)], thereby resulting in a wide-band digital phase shifter [21]–[23]. In this case, the MEMS bridge is placed in series with a fixed metal–insulator–metal (MIM) capacitor (C_s), and $C_{u0} < C_s$, where C_{u0} is the bridge capacitance with zero bias voltage. When the bridge is in the up-state position, the capacitive loading is approximately C_{u0}. When the bridge is pulled down completely over the t-line, its capacitance increases by a factor of 30–50 and the effective periodic loading becomes C_s. A capacitance change of 2–4 can be obtained using this technique, and this results in a large phase shift per centimeter. A 2-bit dc–20-GHz distributed phase shifter was demonstrated by Hayden *et al.* using this approach [22].

The same derivation applies to a digital distributed MEMS t-line phase shifter, except that the MEMS bridges contribute a Brownian noise component in the up-state position only. In this case, and for well-designed phase shifters, $C_{u0} = 2(sC_t)$,

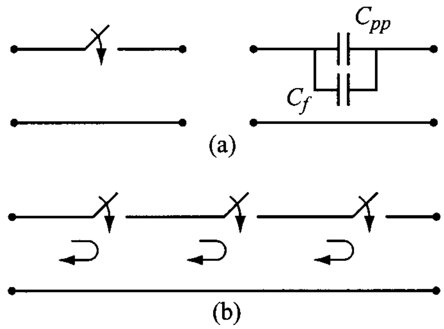

Fig. 6. (a) Series switch with associated up-state capacitance. (b) Reflect-line design.

and the height is around 3 μm above the t-line at zero bias. Also, since C_s is 2–3 C_{u0}, any change in the position of the MEMS bridge (x_n) results in less effect on the total loading of the line due to the series effect of C_s. The phase delay per section is the same as (20) above, except that C_u is replaced by $C_s C_{u0}/(C_s + C_{u0})$. This capacitance is then used in (22) to calculate the phase noise.

The larger MEMS bridge height and the reducing effect of C_s result in less phase noise per section (around -170 dBc/Hz per section). Also, in a digital phase shifter, once the bridges are pulled down, they do not contribute any phase noise since they are fixed in the down-state position. Therefore, the maximum phase noise occurs for the 0° state, when all the MEMS bridges are up. For a 2-bit design, the phase noise is 16×–20× higher than that of a single section, depending on the design.

IX. Brownian Noise in MEMS Series Switches and Phase Shifters

MEMS series switches do not suffer from Brownian noise in the down-state position since the switch is fixed to the substrate. However, in the up-state position, and if used in a reflection-type phase shifter, MEMS series switches do result in a phase-noise component. The reflection phase of a MEMS series switch is

$$\phi = -2\omega C_u Z_0 \tag{24}$$

and when the value of C_u and x_n in (2) and (4) are used, the phase noise relative to the carrier due to the *reflection* from the MEMS series switch becomes

$$P_{\text{ph}} = \frac{1}{2}\,\frac{1}{(1+\gamma)^2}\,\frac{\overline{x_n^2}}{g_0^2}\,\phi_0^2\ /\text{Hz}. \tag{25}$$

Notice that $C_u = 2$–6 fF in most dc-contact switches, and $\gamma = 2$–3 since a large portion of the up-state capacitance of a MEMS series switch is due to the parasitic capacitance between the t-lines and not to the parallel-plate capacitance above the contact points.

MEMS series switches are used in many different phase shifter designs, such as: 1) switched t-lines [24]; 2) reflection type; or 3) reflection type with 3-dB couplers. The switched t-line designs result in insignificant phase noise since the MEMS switches must be in the closed position to pass the energy in the different delay sections. Most MEMS series switches, even if based on cantilever designs, have $k > 5$ N/m and $g_0 > 1.5$ μm. The phase noise of a MEMS series switch (in reflection mode) is, therefore, 100×–300× less than that of a shunt capacitance switch (in transmission mode).

X. Effect of Acceleration and Acoustic Noise

The force due to acceleration noise is

$$f_n = \mathrm{m}a\ \left(\mathrm{N}/\sqrt{\mathrm{Hz}}\right) \tag{26}$$

where m is the mass of the bridge and a is the acceleration noise given in m/s$^2/\sqrt{\text{Hz}}$. In some cases, the units of a are given in g/$\sqrt{\text{Hz}}$, where g is 9.8 m/s^2.

The force due to an acoustic pressure-wave noise is

$$f_n = PA\ \left(\mathrm{N}/\sqrt{\mathrm{Hz}}\right) \tag{27}$$

where A is the area of the MEMS bridge (or cantilever) and P is the differential pressure noise between the top and bottom sides of the MEMS bridge, and is given in Pa/$\sqrt{\text{Hz}}$. For clarification, a 0-dB sound pressure level (SPL) is equivalent to 20 μPa, and a pressure wave of 1 Pa results in a 94-dB SPL, which is a loud audible noise at 500–2000 Hz [25].

The noise forces in (25) and (26) can be used in (1) instead of the thermal noise force ($f_n = \sqrt{4k_BTb}$), and the analysis of the phase noise due to acceleration and acoustic noise is the same as was derived for the Brownian noise for shunt and series elements. For comparison purposes (see below), the force due to thermal noise is around 7.2×10^{-13} N/$\sqrt{\text{Hz}}$ for $k = 10$ N/m, $Q = 1$, and $f_0 = 100$ kHz.

The phase noise due to acceleration and acoustic noise depends on k^{-2} and g_0^{-2} (and not on g_0^{-5}, as in the case thermal noise). The phase noise has the same spectrum as the acceleration/acoustic noise, but drops at -40 dB/dec after the mechanical resonant frequency of the MEMS element. Also, all MEMS bridges (or cantilevers) in the circuit are subjected to the same acceleration/acoustic noise at the same time. Therefore, the resulting noise adds *coherently*, and if N shunt capacitive elements are in the t-line path, then the output phase noise increases by N^2. This is especially important in distributed phase shifters since $N = 16$–24, depending on the design.

Acceleration Noise: The acceleration noise force on a gold bridge with dimensions of $L = 300$ μm, $w = 80$ μm, $t = 0.9$ μm ($m = 4.2 \times 10^{-10}$ kg) and an acceleration noise of 0.01 g/$\sqrt{\text{Hz}}$ perpendicular to the bridge (resulting in a $\pm$1-g acceleration noise over a 10-kHz bandwidth) is $f_n = 4.2 \times 10^{-12}$ N/$\sqrt{\text{Hz}}$. This is around 6× larger than the thermal noise force and results in 16 dB more phase noise per MEMS element. If an aluminum bridge is used, the phase noise is reduced by 17 dB due to the reduced mass of the bridge ($\rho_{\text{gold}}/\rho_{\text{Al}} = 19.3/2.7 = 7.2$). Most cantilever designs have a mass of 10^{-10} to 10^{-11} kg and, therefore, yield similar acceleration noise forces. This leads to the conclusion that phase noise due to acceleration is negligible up to 10 g.

Acoustic Noise: The acoustic noise force on a bridge with $L = 300$ μm and $w = 80$ μm and an acoustic noise of 0.01 Pa/$\sqrt{\text{Hz}}$ (resulting in 94-dB SPL for a 10-kHz bandwidth) is $f_n = 8 \times 10^{-11}$ Pa/$\sqrt{\text{Hz}}$. The acoustic noise force is around 100× higher than the thermal noise force and results in around 40 dB more phase noise per MEMS element. The acoustic noise is the same for gold, Al, or dielectric bridges and cantilevers. The only difference is the area of the MEMS element, which can be 3×–8× smaller for cantilever-based designs. It is, therefore, imperative that MEMS-based circuits be shielded from acoustic noise using packaging techniques. From the calculations above, the phase noise due to acoustic noise is negligible up to a 74-dB SPL.

The effect of the long-term pressure variation in the atmosphere, which is ±5 kPa, is minimal. The reason is that this occurs over hours and days, and the air under the bridge (or cantilever) will equalize to the same pressure above the bridge.

XI. Effect of Bias Voltage Noise

The electrostatic force on a MEMS element is given by

$$F = \frac{\epsilon_0 A}{2g_0^2} V^2 \text{ (N)} \tag{28}$$

where g_0 is the gap height between the electrode and the MEMS element and A is the area of the pull-down electrode. Consider a MEMS element with a dc voltage of $V_{\rm dc}$ and a white noise voltage of v_n (in V/$\sqrt{\rm Hz}$) present on the bias line. The total noise voltage on the bias line is $v_n\sqrt{B}$, where B is the noise bandwidth. The electrostatic force becomes

$$F = \frac{\epsilon_0 A}{2g^2}\left(V_{\rm dc}^2 + 2V_{\rm dc}v_n\sqrt{B} + v_n^2 B\right) \text{ (N).} \tag{29}$$

The mean electrostatic force is

$$\overline{F} = \frac{\epsilon_0 A}{2g^2}\left(V_{\rm dc}^2 + \overline{v_n^2}B\right) \text{ (N).} \tag{30}$$

The noise voltage slightly increases the average pull-down force due to the V^2 effect in (27). This is the steady-state bias point of the MEMS bridge (or cantilever) and as long as the force is less than the pull-down force, there is a stable solution with a gap g ($g < g_0$) [26].

The variance in the electrostatic force is the estimation of the difference between the force and its mean value and is

$$\begin{aligned}\sigma^2 &= E\left[F - \overline{F}\right]^2 \\ &= \left(\frac{\epsilon_0 A}{2g^2}\right)^2 E\left[2V_{\rm dc}v_n\sqrt{B} + B\left(v_n^2 - \overline{v_n^2}\right)\right]^2 \\ &= \left(\frac{\epsilon_0 A}{2g^2}\right)^2 \left(4V_{\rm dc}^2 v_n^2 B + E\left[B^2\left(v_n^2 - \overline{v_n^2}\right)^2\right]\right)\end{aligned} \tag{31}$$

where $E[x]$ is the estimation function of x. For white noise with a Gaussian spectral power density (SPD), the second term reduces to $2B^2(\overline{v_n^2})^2$. Therefore, for a dc voltage applied on the MEMS element, as in the case of varactor applications, the noise force (fluctuations around the mean) is

$$f_n = \sqrt{\sigma^2} = \frac{\epsilon_0 A}{2g^2}(2V_{\rm dc}v_n)\sqrt{B} \text{ (N)} \tag{32}$$

and for no dc-bias voltage, as in the case of MEMS switches in the up-state position, the noise force is

$$f_n = \frac{\epsilon_0 A}{2g^2}\left(\sqrt{2v_n^2}\right) B \text{ (N).} \tag{33}$$

This is the total force on the MEMS element for a bandwidth B. If the force is required in N/$\sqrt{\rm Hz}$, then the force in (31) and (32) must be divided by $\sqrt{B}$. For a MEMS varactor with $V_{\rm dc} = 20$ V, $g = 2$ μm, an electrode area of 100 × 100 μm^2, and $v_n = 0.1$ mV/$\sqrt{\rm Hz}$ (equivalent to 33 mV of noise over a bandwidth of 100 kHz), $f_n = 5 \times 10^{-11}$ N/$\sqrt{\rm Hz}$. This is 70× larger than the thermal noise force, and results in 37 dB more phase noise than the Brownian noise level. Therefore, it is imperative that the bias noise on varactors be tightly controlled. (This also applies to distributed *analog* phase shifters).

For a MEMS switch with $V_{\rm dc} = 0$ V, $g = 3$ μm, an electrode area of 100 × 100 μm^2, and $v_n = 0.1$ mV/$\sqrt{\rm Hz}$ over a 100-kHz bandwidth, then $f_n = 2.2 \times 10^{-14}$ N/$\sqrt{\rm Hz}$, and is less than the Brownian noise force. If the bias noise voltage increases to 1 mV/$\sqrt{\rm Hz}$ (330 mV over a 100-kHz bandwidth), the noise force increases to 2.2 × 10^{-12} N/$\sqrt{\rm Hz}$, and results in 10 dB more phase noise than the Brownian noise component. Therefore, it is essential that the noise voltage be kept lower than 200–300 mV over the integration bandwidth, and the dc-bias voltage be kept at 0 V when the switch is in the up-state position.

Finally, if several MEMS bridges (or cantilevers) are used, then the total output noise is added coherently since the bias lines are all placed close to each other and "pick-up" the same noise waveform. Switching noise waveforms are not generally white noise, and their SPD must to known to accurately predict the resulting phase-noise levels. However, the contribution will be very low if the SPD is lower than an equivalent 100 mV of white noise over the integration bandwidth.

XII. Conclusion

This paper detailed the phase-noise analysis in MEMS-based circuits phase shifters. It is seen that if the MEMS shunt switch is well designed ($Q > 0.5$, $k > 10$ N/m, $f_0 > 50$ kHz), then it will result in a truly negligible phase noise from thermal effects (Brownian noise). The phase noise is so low that it is hard to measure using even the best phase-noise measurement equipment at 10 GHz. However, low-k shunt switches, and/or switches which are suspended at low gap heights (1.5 μm or less) do result in a 20–40 dB higher phase-noise component. Also, varactor-based phase shifters result in a relatively high phase noise since all of the capacitance (and its movement) is used in the design. Distributed phase shifters, where the bridge capacitance is added to the t-line capacitance, result in a phase noise that is around 20 dB lower than varactor-based design (but still 20 dB higher than switched network designs). Also, series switches result in virtually no phase noise in the reflect-mode since their up-state capacitance is extremely low. The effect of acceleration, acoustic, and bias voltage noise can be calculated in a very similar fashion to the Brownian noise. It was found that their contribution is quite low for an acceleration noise of 10 g or less, an acoustic SPL of 74 dB or less, and a voltage bias noise of 0.3 V or less.

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